Balance-of-Payments Shocks and Central-Bank Swap Lines in a Small Open Economy

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Abstract

The first goal of this paper is to analyze output, consumption, and exchange-rate effects of belief shocks regarding future changes in the asset market structure in a small open economy. Specifically, an exogenous temporary increase in the demand for international currencies signals the possibility of a future permanent change in the asset market structure such as a shift from complete markets to financial autarky. The transition then endogenously arises with an adjustment of the home country's net foreign asset position. In this case, the government's holding of international reserves can help mitigate the potential impact of this balance-of-payments shock on the nominal exchange rate and the aggregate output. The second goal of this paper is to show that when the government's holding of international reserves is not sufficiently large, the introduction of central-bank swap lines (used during the recent global financial crisis) can help defend the economy from adverse effects of the balance-of-payments shocks signaling a future permanent change in the asset market structure. It is also shown in this paper that standard debt contracts between the Federal Reserve and foreign central banks are inferior to swap contracts between them in the presence of the informational advantage of foreign central banks for local financial institutions in their jurisdictions.

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1 Introduction

Recent years (nearly two decades) have witnessed a relatively long trend of reserve accumulation prior to the Global Financial Crisis and its substantial declines during the Crisis in the group of emerging-market countries. It had been remarked in academia and among practical policy makers before the Crisis that their hoarding of international reserves might be excessive in light of the so-called "Guidotti-Greenspan" prescription for reserve adequacy, especially for reserve stocks far more than enough to cover potential disruptions in the international settlements of imports and even in the roll-over of their short-term external debts. During the Crisis, the Federal Reserve provided foreign-exchange swap lines with other central banks including the European Central Bank, Swiss National Bank, the Bank of Japan, and the Bank of Korea.

The first goal of this paper is to analyze output, consumption, and exchange-rate effects of belief shocks regarding future changes in the asset market structure in a small open economy. Specifically, an exogenous temporary increase in the demand for international currencies signals the possibility of a future permanent change in the asset market structure such as a shift from complete markets to financial autarky. The transition then endogenously arises with an adjustment of the home country's net foreign asset position. In this case, the government's holding of international reserves can help mitigate the potential impact of this balance-of-payments shock on the nominal exchange rate and the aggregate output.

To motivate the introduction of the balance-of-payments shocks into a small-open economy, it would be worthwhile to discus the view of Obstfeld, Shambaugh, and Taylor (2009 and 2010) that a central bank holds its international reserves as protection against "double-drain" crisis scenarios in which banking and currency problems interact in ways likely to cause sharp and disruptive external currency depreciation. Based on this crisis scenario, an exogenous temporary increase in the demand for international currencies (defined as a balance-of-payments shock) could arise under capital flights during the "double-drain" crisis. The possibility of a future shift from complete asset markets into financial autarky reflects a pessimistic expectation about future financial conditions in the onset of the "double-drain" crisis. The actual consequence of this belief is that an endogenous change in the structure of asset market can take place at the current period.

Both output and consumption can drop when households and firms begin to believe that the change in the asset market structure would raise future inflation rates. Moreover, their beliefs could be confirmed by observing a rise in the current CPI inflation rate and a sharp depreciation of the nominal exchange rate when the economy is hit by an adverse balance-of-payments shock. In sum, these depressionary effects of an adverse balance-of-payments shock arise because of an endogenous shift in the asset market structure and anticipated future output costs of high future inflation.

The second goal of this paper is to show that when the government's holding of international reserves is not sufficiently large, the introduction of central-bank swap lines (used during the recent global financial crisis) can help defend the economy from adverse effects of the balance-of-payments shocks signaling a future permanent change in the asset market structure. A standard debt contract under asymmetric information between lenders and borrowers may not work to model actual swap lines between the Federal Reserve and foreign central banks. The main reason for this argument is that although debt contact models require lenders to take credit risks, the Federal Reserve did not take credit risks that might arise from its swap lines. It is shown in this paper that standard debt contracts between the Federal Reserve and foreign central banks are inferior to swap contracts between them in the presence of the informational advantage of foreign central banks for local financial institutions in their jurisdictions.

Specifically, the model of central bank swap lines developed in this paper reflects the fact that the Federal reserve delegates its lending capacity to foreign central banks in order to save intermediation costs, which might arise with the asymmetric information about balance sheet conditions of local financial institutions who benefited from foreign exchange swap lines. Hence, foreign central banks engage in the financial intermediation between the Federal Reserve and non-US financial institutions. Specifically, foreign central banks distribute U.S. dollars to local financial institutions with "variable-rate" auctions, while they make currency swap contracts with the Federal Reserve.

However, allowing for the asymmetric information between the Federal Reserve and foreign central banks still requires a country to accumulate international reserves as collateral for its swap contracts. As a result, it would make sense to define the financial capacity of international reserves as the sum of its international reserves and the central bank's borrowing from central bank swap lines, which can be used to attain the stability of domestic financial markets and foreign exchange markets. In this framework, therefore, central bank swap lines act as a mechanism to create and also increase the financial capacity of international reserves.

The rest of this paper is organized as follows. The next section present a model of small-open economies that can be subject to balance-of-payments shocks. Section 3 describe a model of central bank swap lines that are intended to mitigate the shortage of U.S. dollars in local financial markets. In section 4 summarizes the role of central bank swap lines in the small open economy that is inflicted by adverse balance-of-payments shocks. Section 5 concludes.

2 A Small Open Economy Model

There are two countries, called H (Home)and F (Foreign), in the world where a fraction of agents $[0; \quad n)$ of unit mass lives in country H and the other fraction $(n; \quad 1]$ belongs to country F. A continuum of differentiated goods exists in the world. Each type of these tradable goods is produced by either country H or country F, while each country produces a number of different brands with measure equal to population size.[1] The preferences at period 0 of the representative household in country H is represented by the following function:

$$\sum_{t=0}^{\infty} E_0[U(C_t, \zeta_{C,t}) + L(M_t/P_t, \zeta_{M,t}) - V(N_t, \zeta_{N,t})]$$

where C_t is the aggregate consumption index at period t, M_t is the nominal balances at period t, P_t is the aggregate price index at period t, N_t is the number of hours worked at period t. The functional forms of utility functions for consumption, real balances, and the amount of hours worked are given by

$$U(C_t, \zeta_t) = \zeta_{C,t}\frac{C_t^{1-\sigma_C} - 1}{1-\sigma_C}; \;\; L(M_t/P_t, \zeta_{M,t}) = \zeta_{M,t}\frac{(M_t/P_t)^{1-\sigma_M} - 1}{1-\sigma_M}; \;\; V(N_t, \zeta_{N,t}) = \frac{\zeta_{N,t}^{-\sigma_N} N_t^{1+\sigma_N}}{1+\sigma_N}$$

where preference shocks are represented by $\zeta_{C,t}$, $\zeta_{M,t}$, and $\zeta_{N,t}$.

Each household solves two cost minimization problems to derive demand functions for home and foreign differentiated goods:

$$\min \int_0^n P_{i,t}(z)C_{i,t}(z)dz \;\; \text{s.t.} \;\; C_{i,t} = [n^{-1/\epsilon}\int_0^n C_{i,t}(z)^{\frac{\epsilon-1}{\epsilon}}dz]^{\frac{\epsilon}{\epsilon-1}}$$

for $i = H$ and F and where $\epsilon > 1$ and their sub-price indices are given by

$$P_{H,t} = [n^{-1/\epsilon}\int_0^n P_{H,t}(z)^{1-\epsilon}dz]^{\frac{1}{\epsilon-1}}; \;\; P_{F,t} = [(1-n)^{-1/\epsilon}\int_n^1 P_{F,t}(z)^{1-\epsilon}dz]^{\frac{1}{\epsilon-1}}$$

In addition, the consumption basket in the utility functions of home residents is defined as

$$C_t = (a_H^{1/\theta}C_{H,t}^{\frac{\theta-1}{\theta}} + (1-a_H)^{1/\theta}C_{F,t}^{\frac{\theta-1}{\theta}})^{\frac{\theta}{\theta-1}}$$

The corresponding consumer price index (CPI) is then given by

$$P_t = (a_H P_{H,t}^{1-\theta} + (1-a_H)P_{F,t}^{1-\theta})^{\frac{1}{\theta-1}}$$

The aggregate consumption index for the rest of the world is

$$C_t^* = ({a_H^*}^{1/\theta}(C_{H,t}^*)^{\frac{\theta-1}{\theta}} + (1-a_H^*)^{1/\theta}(C_{F,t}^*)^{\frac{\theta-1}{\theta}})^{\frac{\theta}{\theta-1}}$$

It is also assumed that $a_H = 1$ - $(1-n)\lambda$ and $a_H^* = n\lambda$ where λ measures the degree of openness, following Sutherland (2005) and De Paoli (2009). The usual technique employed in the literature

[1] The modification of a two-country model into a small-open economy model has been widely used in the literature such as Gali and Monacelli (2005), De Paoli (2009), and Farhi and Werning (2013a and 2013b). The two-country version of this section' model has been used in the analysis of Corsetti, Dedola, and Leduc (2010) for optimal monetary policy.

to turn this model into a small open economy is to take the limit for $n \to 0$ in the definitions of a_H and a_H^* after deriving both price indices and consumption indices. The resulting demand function of firm h in the home country is given by

$$Y_t(h) = (\frac{P_{H,t}(h)}{P_{H,t}})^{-\epsilon} Y_t$$

where Y_t denotes the aggregate demand at period t:

$$Y_t = (\frac{P_{H,t}}{P_t})^{-\theta}((1-\lambda)C_t + \lambda Q_t^{\theta} C_t^*) + G_t.$$

The consumption price index also turns out to be

$$P_t = ((1-\lambda)P_{H,t}^{1-\theta} + \lambda P_{F,t}^{1-\theta})^{\frac{1}{\theta-1}}.$$

The ratio of GDP deflator to CPI is

$$(P_{H,t}/P_t)^{1-\theta} = (1 - \lambda Q_t^{1-\theta})/(1-\lambda)$$

In addition, the aggregate resource constraint at period t can be written as

$$P_{H,t}(Y_t - G_t) = P_t(C_t + \mathrm{NX}_t)$$

where NX_t is the real net exports measured in the unit of consumption goods. The real net exports can be then written as a function of the consumption ratio between two countries and the real exchange rate:

$$\mathrm{NX}_t = -\lambda Q_t^{1-\theta}(C_t - \frac{1-\lambda Q_t^{1-\theta}}{1-\lambda} Q_t^{2\theta-1} C_t^*).$$

The flow budget constraint of an individual household in period t can be written as

$$M_t + \frac{B_{H,t}^H}{1+i_{H,t}} + \frac{S_t B_{F,t}}{1+i_{F,t}} \leq B_{H,t-1}^H + S_t B_{F,t-1} + M_{t-1} + W_t N_t - P_t C_t - P_{H,t} T_t - \Phi_t$$

where $B_{H,t}^H$ and $B_{F,t}$ denote holdings of domestic- and foreign-currency denominated nominal bonds with their interest rates denoted by $i_{H,t}$ and $i_{F,t}$ respectively, $C_{H,t}$ and $C_{F,t}$ are demands for home and foreign goods with their domestic prices denoted by $P_{H,t}$ and $P_{F,t}$ respectively, W_t is the nominal wage, N_t is the number of hours worked at period t, T_t is the real tax in the unit of GDP, Φ_t is the nominal profit at period t. The optimization conditions of the representative household for their bond holdings can be written as

$$\Lambda_t = (1 + i_{H,t})\beta E_t[\frac{\Lambda_{t+1}}{\Pi_{t+1}}]$$

$$\Lambda_t = (1 + i_{F,t})\beta E_t[\frac{\Lambda_{t+1} S_{t+1}}{S_t \Pi_{t+1}}]$$

where Λ_t represents the marginal utility of consumption at period t and Π_t is the CPI inflation rate at period t.

Domestic residents hold domestic-denominated money balances, whereas they do not accumulate foreign-currency denominated money balances. Similarly, foreign investors do not accumulate the domestic-denominated money balances, while they can hold the home government's securities. While the home government issues nominal one-period risk-less securities, the market-clearing condition of these securities is given by $B_{H,t} = B_{H,t}^H + B_{H,t}^F$ where $B_{H,t}$ is their total outstanding issue at period t and $B_{H,t}^F$ is the nominal value of bond portfolios held by foreign investors. The government's flow budget constraint at period t is given by

$$M_t - M_{t-1} + \frac{B_{H,t}}{1+i_{H,t}} = B_{H,t-1} + S_t(I_t^* - I_{t-1}^*) + P_t(T_t - G_t)$$

where I_t^* is the dollar value of international reserves held by the government at the end of period t, T_t is the real amount of lump-sum taxes at period t, and G_t is the government's real expenditures at period t.

Having defined budget constraints of the government and the representative household, the excess demand at period t for U.S. dollars in the foreign exchange market is given by

$$D_{F,t} = S_t(\frac{B_{F,t}}{1+i_{F,t}} - B_{F,t-1}) + (B_{H,t-1}^F - \frac{B_{H,t}^F}{1+i_{H,t}}) - P_t\mathrm{NX}_t + \Psi_t$$

where $D_{F,t}$ denotes the excess demand at period t of U.S. dollars in the unit of the home currency, NX_t is the real value of net exports at period t, and $B_{H,t-1}^F$ is the nominal foreign debts carried over from period $t-1$. The first parenthesis in the right hand side of this equation reflects the flow demand of U.S dollars that are created by domestic investors for international bonds. The second parenthesis is the dollar- denominated repayment of foreign debts subtracted from new roll-overs made at period t. The third term summarizes contributions of cross-border goods trades to the aggregate excess demand of U.S. dollars in the foreign exchange market. The last term reflects an exogenous component of the excess demand of U.S. dollars. For example, unfounded fears of future large depreciations of the home currency can lead to capital flights, which might contribute to an exogenous increase in the demand for U.S. dollars.

Turning to the definition of the balance-of-payments account, the current account measures the monetary value of an actual export or import of a good or service, while the capital and financial account measures monetary flows between countries used to purchase financial assets such as stocks, bonds, real estate and other related items. The sum of current and capital accounts equals zero. Therefore, a country that runs a current account trade deficit will have an offsetting capital account trade surplus. It is notable that the capital and financial account includes official reserve transactions. The government can affect the exchange rate by buying and selling its international reserves, while the accumulation and de-accumulation of international reserves is recorded in the account of official reserve transactions. As a result, the sum of changes in international reserves and the excess demand (denoted by $D_{F,t}$) should be zero at an equilibrium.

Having discussed the balance-of-payment equation, I now define a balance-of-payment shock as an exogenous increase in Ψ_t shown above. This exogenous shock also makes people believe that the economy plunges into financial autarky in the future. Specifically, agents learn an unexpected increase in Ψ_t at period t and also anticipate that $\mathrm{NX}_{t+1} = 0$ holds as the economy enters into financial autarky at period $t+1$. The requirement of this belief to be model-consistent leads to an adjustment of the home country's asset position at period t. In order to compute the resulting equilibrium adjustment of the country's asset position, optimization conditions of bond holdings are substituted into the balance-of-payment equation. The resulting balance-of-payment equation can be written as

$$z_t = \beta E_t[\frac{\Lambda_{t+1}}{\Lambda_t} z_{t+1}] - (\frac{D_{F,t}}{P_t} + \mathrm{NX}_t - \frac{\Psi_t}{P_t})$$

where Λ_t is the marginal utility consumption at period t and z_t ($\equiv Q_t B_{F,t-1}/P_t^* + B_{F,t-1}^H/P_t$) is the real value at period t of the previous period's net foreign asset position. In addition, the balance-of-payment equation at period $t+1$ turns out to be

$$z_{t+1} = \beta E_{t+1}[\frac{\Lambda_{t+2}}{\Lambda_{t+1}} z_{t+2}] - (\frac{D_{F,t+1}}{P_{t+1}} + \mathrm{NX}_{t+1} - \frac{\Psi_{t+1}}{P_{t+1}})$$

The balance-of-payment shock is purely transient so that $\Psi_t = \Psi$ (> 0) and $\Psi_{t+k} = 0$ for $k = 1, 2, \cdots, \infty$. Moreover, the assumption of financial autarky at period $t+1$ leads to $\mathrm{NX}_{t+k} = 0$ and $D_{F,t+k}$ for $k = 1, 2, \cdots, \infty$. Agents at period t thus anticipate that $z_{t+1} = 0$ holds at period $t+1$. Therefore, the balance-of-payment equation at period t under the belief formed at the beginning of period t can be rewritten as

$$z_t = -(\frac{D_{F,t}}{P_t} + \mathrm{NX}_t - \frac{\Psi_t}{P_t}).$$

It would be worthwhile to discuss the motivation of this balance-of-payments shocks in a small-open economy. As mentioned in the introduction, a central bank holds its international reserves as protection against "double-drain" crisis scenarios in which banking and currency problems interact in ways likely to cause sharp and disruptive external currency depreciation. The exogenous temporary increase in the demand for international currencies is a consequence of capital fights that can be observed in the "double-drain" crisis. It might be also plausible to allow for the possibility of an endogenous shift from complete asset markets into financial autarky as a consequence of the "double-drain" crisis.

The size of $D_{F,t}$ reflects the government's intervention in the foreign exchange market. For example, the absence of the government's intervention requires that $D_{F,t} = 0$ should hold at an equilibrium. If there is any intervention in the foreign exchange market, it should change the government's holding of international reserves:

$$I_t^* - I_{t-1}^* = -\frac{D_{F,t}}{S_t}.$$

The magnitude of the government's intervention at period t is bounded by its beginning-of-period holding of international reserves (denoted by I_{t-1}^*). In sum, the substitution of net-exports equation into the balance-of-payment equation shown above leads to an equilibrium condition

$$C_t = \frac{1 - \lambda Q_t^{1-\theta}}{1-\lambda} Q_t^{2\theta-1} C_t^* - \lambda^{-1} Q_t^{-\theta} K_t$$

where K_t is a state variable that reflects the effect of an exogenous shift in the balance-of-payment, the government's policy choice, and the home country's lagged net asset position:

$$K_t = Q_t^{-1}(\frac{\Psi_t - D_{F,t}}{P_t} - z_t).$$

The range of K_t is given by

$$\frac{\Psi_t}{P_t^* S_t} - \frac{I_{t-1}^*}{P_t^*} - \frac{z_t}{Q_t} \leq K_t \leq \frac{\Psi_t}{P_t^* S_t} - \frac{z_t}{Q_t}$$

It follows from these inequalities that the government's holding of international reserves helps insulate the home country from the potential adverse impact of the balance-of-payment shock. For example, it might be possible to have $K_t = 0$ if the government's holding of international reserves is large enough and the balance-of-payment shock is sufficiently small. In addition, a negative net foreign asset in the previous period as well as the size of Ψ_t shifts upward the range of values for K_t at period t.

Turning to the balance-of-payments equation, it should be noted that this equation does not create an effective binding equilibrium restriction unless there is any restriction on the conditional expectation term in the equation such as $z_{t+1} = 0$. For this reason, if the government's holding of international reserves is large enough, it might want to use its international reserves to control the public'e expectation regarding future change in the asset market structure. In this case, if the government were to intervene persistently in the foreign exchange market, it is possible to compute the time period when it uses up international reserves. For example, let us suppose that $I_T^* = 0$ for some $T > 0$. So we have $I_{T-1}^* = D_{F,T}/S_T$ for a given positive value of $D_{F,T} > 0$. Hence, a successive backward iteration of the accumulation equation of international reserves shown above leads to the following equation:

$$I_{-1}^* = S_0 \sum_{t=0}^{T} (\frac{S_0}{S_t}) D_{F,t}$$

Table 1: Balance-of-payments shock with an expectation of a future permanent change in asset market structure

Exchange-Rate Adjustment	$\rightarrow$	**Asset Adjustment**	$\rightarrow$	**Financial Autarky**
Depletion of International Reserves		Adjustment of Asset Positions		Absence of Cross-Border Financial Transactions
$I^*_{T-2} \leq D_{F,T-1}/S_{T-1}$		$D_{F,T} = 0$		$D_{F,T+k} = 0$ $\text{NX}_{T+k} = 0$ for k =1, 2, $\cdots$
Original Structure for Asset Markets	$\rightarrow$	Transition Period for a New Structure	$\rightarrow$	A New Structure of Asset Markets

Given an initial value of international reserves, this equation can be used to determine a value of T (the time point at which the complete depletion of international reserves takes place) endogenously if a set of expected deprecation rates and a set of expected future excess demands are provided. During the time periods when government can use its international reserves to defend the original asset market structure, only exchange-rate adjustments can take place. In this case, the balance-of-payments equation does not produce an effective binding equilibrium restriction as mentioned above. But if the government's holding of international reserves is not large enough to defend, it is necessary to have an adjustment of its net foreign asset position that reflects the public's expectation that the economy will enter into financial autarchy. Table 1 summarizes how this endogenous change in the asset market structure proceeds.

In order to close the model, domestic firms are assumed to set both their local and international prices in the unit of the domestic currency (so called producer's currency pricing) according to the Calvo pricing model. In this model, a fraction of firms reset their price during each period, while the other fraction do not. Each firm that resets its price chooses its price by maximizing the following expected present-value of profits:

$$\sum_{k=0}^{\infty}(\alpha\beta)^k E_t[\Lambda_{t+k}(\frac{P^*_{H,t}}{P_{H,t+k}})^{-\epsilon}Y_{t+k}(\frac{P^*_{H,t}}{P_{H,t+k}} - \frac{W_{t+k}}{P_{H,t+k}})]$$

where α is the fraction of firms that reset their prices in each period and $P^*_{H,t}$ is the optimal reset price at period t. The first-order conditions for this profit maximization can be represented by a set of recursive equations:

$$z_{1t} = C_t^{-\sigma_C}Y_t + \alpha\beta E_t[\Pi^{\epsilon-1}_{H,t+1}z_{1t+1}]$$

$$z_{2t} = N_t^{\sigma_N}\frac{W_tY_t}{P_{H,t}} + \alpha\beta E_t[\Pi^{\epsilon}_{H,t+1}z_{2t+1}]$$

$$\frac{P^*_{H,t}}{P_{H,t}} = \frac{\epsilon}{\epsilon - 1}\frac{z_{1t}}{z_{2t}}$$

The GDP deflator inflation of domestic foods can be determined by the following equation:

$$1 = (1-\alpha)(\frac{P^*_{H,t}}{P_{H,t}})^{1-\epsilon} + \alpha \Pi_{H,t}^{\epsilon-1}.$$

The aggregate production function also can be written as

$$Y_t = N_t/\Delta_t$$

where Δ_t is the relative price distortion at period t:

$$\Delta_t = (1-\alpha)(\frac{P^*_{H,t}}{P_{H,t}})^{-\epsilon} + \alpha \Pi_{H,t}^{\epsilon}\Delta_{t-1}$$

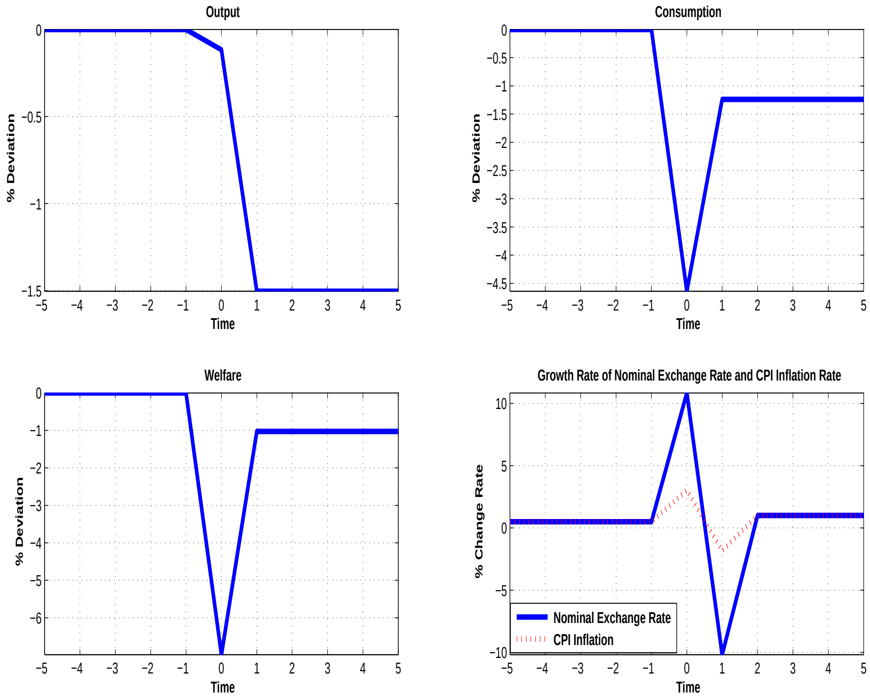
It follows from this equation that the relative price distortion rises with the GDP deflator inflation. It would be worthwhile to mention that when agents anticipate a higher inflation under financial autarky, their forecasts of future financial autarky's output cost arises with expected increases of relative price distortion. Therefore, when an adverse shock to the balance-of-payments shock takes place, it might generate a temporary output fall because of their beliefs that future inflation rates will rise as the economy runs into financial autarky. In sum, a depressionary output effect of the balance-of-payments shock could arise because of the channel through which the public's anticipation regarding future paths of monetary policy variables affects the current economic condition.[2]

Figure 1 shows output, consumption, and exchange-rate effects of the balance-of-payments shock that invokes the public's belief regarding a future change in the asset market structure. Households and firms learn the shock at period 0 and begin to believe that the economy will run into financial autarky at period 1 onward. The expected number of transition periods for the future shift from complete markets to financial autarky depends on the initial amount of international reserves as discussed above. It is assumed in this figure that the government's holding of international reserves is not sufficient to insulate the economy from this balance-of-payments shock. Specifically, the value of K_t is set to be $K_t = 0.02$.

The two upper panels demonstrate that both output and consumption (at period 0) are lower during the transition period than their previous levels. The paths at period 1 onward in these panels are expected paths of output and consumption. Specifically, agents at period 0 expect that the aggregate consumption will recover at period 1 onward, while the aggregate output will be permanently lower in financial autarchy than in complete asset markets. The nominal exchange rate displays a sharp depreciation at period 0 with a rise in the CPI inflation rate. The depressionary

[2] Del Negro, Giannoni, and Patterson (2012) discuss the impact of the public's long-term expectations about future economic conditions on the current economic condition. Chinn and Quayyum (2013) and Kiley (2013) also discuss the effect of forward guidance on the nominal exchange rate that is implied by the uncovered interest parity condition.

Figure 1: Effects of Balance-of-Payments Shocks



Note: The panels of this figure displays effects of the balance-of-payments shock that invokes the public's belief regarding a future change in the asset market structure. The belief takes place at period 0. The value of K_t is set to be $K_t = 0.02$. In addition, $\alpha = 0.75$, $\beta = 0.98$, $\sigma_C = 1$, $\sigma_N = 2$, $\lambda = 0.2$, $\theta = 0.8$, $\epsilon = 11$, and $C^* = 1$.

output effect of this adverse balance-of-payments shock takes place because there is an expectation channel through which the public's anticipation regarding future paths of monetary policy variables affects the current economic condition. In particular, the public's belief that the GDP deflator inflation rate is higher in financial autarky than in complete asset markets is the main mechanism to generate a temporary decline in the aggregate output.

3 A Model of Central-Bank Swap Lines

In this section, I present a model of central bank foreign-exchange swap lines. There are four players in this model: The Federal Reserve, foreign central banks, local whole-sale financial institutions, and retail private banks, who are all risk-neutral. In addition, individual households are not allowed to have access to wholesale loan markets discussed below.

The foreign central bank holds a total stock of its international reserves I_t^* at the beginning of period t. The foreign central bank withdraw an amount of international reserves F_t^* from its total balance of reserves at the time when it enters into the foreign exchange swap contract with the

Federal Reserve. By using this contract, it adds (A_t^* - F_t^*) to its own holding of reserves to make a total stock of A_t^* available at period t. In return for its borrowing, the foreign central bank will pay $(1 + z_t)(A_t^* - F_t^*)$ to the Federal Reserve where z_t is the contractual interest rate. It should be noted that allowing for both swap reserves and non-swap reserves helps explain how the reference nominal exchange rate (denoted by S_t) is determined through interactions of different wholesale banks.[3]

The important feature of central bank swap lines is that the Federal Reserve does not bear any credit risk associated with the distribution of U.S. dollars drawn from central bank swap lines to local banks. The Federal Reserve is not a counter-party to the loan extended by the foreign central bank to local depository institutions. Hence, the foreign central bank bears the credit risk associated with the loans that it makes to institutions in its jurisdiction.

Another important feature of central bank swap lines is the repurchased agreement between the Federal Reserve and foreign central banks. In the beginning of their transaction, the foreign central bank sells its domestic currency M_t to the Federal Reserve in exchange for dollars $(A_t^* - F_t^*)$ at a spot nominal exchange rate S_t, so that $M_t = S_t(A_t^* - F_t^*)$ should hold. The Federal Reserve and the foreign central bank also enter into an agreement that requires the foreign central bank to buy back its currency at the same exchange rate S_t. The use of the same exchange rate for the repurchase agreement described above implies that the recorded value of the foreign currency amount is not affected by changes in the market exchange rate, and the Federal Reserve bears no exchange rate risk.

The foreign central bank uses "variable-rate auctions" when it provides loans with financial institutions in its jurisdiction. It means that U.S. dollar loans are allotted at the rate submitted by each successful bidder whose bid is greater than the minimum rate set by the central bank. In order to rationalize this assumption, it is worthy of noting that the European Central Bank, the Swiss National Bank, and the Bank of England all conducted overnight variable-rate auctions for U.S. dollars during the swap line program's second and third phases of foreign swap lines between the Federal Reserve and theses central banks. The rates bid for dollars in these auctions tend to reflect pressures in overseas U.S. dollar funding markets over this period.[4]

In order to rationalize the establishment of central bank swap lines, it is assumed that there are

[3] William and Moessner (2010) and Fleming and Klagge (2010) describe how central-bank swap lines had been established and used during the recent global financial crisis. Aizenman and Lee (2010), Aizenman, Jinjarak and Park (2011), and Dominguez, Hashmoto and Ito (2012) have discussed various issues of the recent accumulation of international reserves by emerging-market countries.

[4] The variable-rate tender is defined as a tender procedure whereby counter-parties bid both the amount of money they want to transact with the central bank and the interest rate at which they want to enter into the transaction in the glossary of the European Central Bank. In contrast, a fixed rate tender is defined as a tender procedure where the interest rate is specified in advance by the central bank and in which participating counter-parties bid the amount of money they want to transact at that interest rate.

Table 2: U.S. Dollar Liquidity Shocks in Local Retail Financial Markets

Retail Banks in Local Market i
Demands for U.S. dollars: $D^i_{F,t}$ in local currencies

$\searrow$

Local Market Equilibrium: $D^i_{F,t} = \omega^i_t S_t A^*_t$
ω^i_t reflects the shortage of U.S. dollars in local market i given a supply level of $S_t A^*_t$.
No Arbitrage Condition in Local Market i: $1 + i^{*,i}_t = (1 + i_t)\omega^i_t S_t / f_t$
where $i^{*,i}_t$ is the interest rate for U.S. dollar-denominated loans in local market i.

$\nwarrow$

Wholesale Bank i
To distribute U.S. dollar loans to local market i
To bid an amount of A^*_t for central bank's auction

$\swarrow$

Aggregate Market Clearing Condition
Supply side of the aggregate market
$(1 - \Upsilon(\bar{\omega}_t))A^*_t$ via central bank's auctions
$\Upsilon(\bar{\omega}_t)A^*_t$ via normal distribution channels
Market clearing condition $D_{F,t} = S_t A^*_t$
where $D_{F,t} = \int_0^\infty D^i_{F,t}(\omega) d\Upsilon(\omega)$

U.S. dollar liquidity shocks in local financial markets. The magnitude of U.S. dollar liquidity shocks is measured by the excess demand of U.S. dollars in each local market. There is a continuum of local financial markets where retail banks operate to help the exchange between different currencies. Each local market is differentiated ex-post depending on realized values of U.S. dollar liquidity shocks that are idiosyncratic across different local markets. In addition, as mentioned in the introduction, it would be essential to address the issue of moral hazard in the provision of U.S. dollars through central bank swap lines. The potential possibility of moral hazard problems could exist because of the asymmetric information between the Federal Reserve and local institutions. Specifically, the Federal Reserve alone does not observe these shocks unless it pays physical resources, which will be elaborated later.

I now turn to a formal description of liquidity shocks for U.S. dollars in local markets. Let us suppose that retail banks who operate in local market i have their total demands for U.S. dollars of $D^i_{F,t}$ in local currencies. The amount of U.S. dollars that are available from supplies of wholesale banks in local market i is A^*_t, while the spot exchange rate at period t in the wholesale market is S_t. Hence, the market-clearing condition in local market i is $D^i_{F,t} = \omega^i_t S_t A^*_t$ where ω^i_t reflects the shortage of U.S. dollars in local market i given a supply level of A^*_t. The random variable ω is independently and identically distributed across time and across local markets with a continuous and once-differentiable cumulative probability distribution function, $\Upsilon(\omega)$, over a non-negative support and also this expected value is equal to one, $E[\omega] = 1$.

It is also assumed that local banks can enter into forward exchange-rate contracts to hedge exchange risks. The forward rate at period t for the spot nominal exchange rate at period $t+1$ is denoted by f_t. The absence of arbitrage in local market i is then given by $1 + i^{*,i}_t = (1 + i_t)\omega^i_t S_t/f_t$, where $i^{*,i}_t$ is the interest rate for U.S. dollar-denominated loans in local market i. As a result, a wholesale bank who enters into local market i has an ex-post gross return from its loan portfolio of $\omega^i_t\{(1 + i_t)S_t/f_t\}$ where $\{(1 + i_t)S_t/f_t\}$ can be interpreted as the ex-post aggregate average return on domestic loan portfolios. In this case, the aggregate market clearing condition should be $D_{F,t} = S_t A^*_t$ where $D_{F,t} = \int_0^\infty D^i_{F,t}(\omega)d\Upsilon(\omega)$. As a result, the aggregate average gross return of U.S. dollar loans made by wholesale banks is $S_t(1 + i_t)A^*_t/f_t$. However, it should be noted that only a fraction of wholesale banks have U.S. dollar loans from the central bank's auctions. Those banks who are not successful or do not participate in the central bank's auctions not need to make any repayments to the central bank. A diagram that helps clarify the role of market-specific liquidity shocks in this model is included in Table 2.

Definition 3.1 (News Shocks for True Liquidity Shocks) The market-clearing condition in local market i is $D^i_{F,t} = \omega^i_t S_t A^*_t$ where ω^i_t reflects the shortage of U.S. dollars in local market i given

Table 3: Information Flows of Agents and Procedure of Contracts

Initial Sage of Swap Lines
To begin with F_t^*
To set the contractual interest rate (=z_t)
To exchange dollars with domestic currencies
$(A_t^* - F_t^* = M_t/S_t)$

$\Downarrow$

Middle Stage
To observe "news shocks"
To announce a "variable-rate auction"
To distribute $(1 - \Upsilon(\bar{\omega}_t))A_t^*$ via auction
To set the minimum rate for the auction
at the contractual rate of swap lines
$\bar{\omega}_t\{(1 + i_t)S_t/f_t\}A_t^* = (1 + z_t)(A_t^* - F_t^*)$

Domestic Wholesale Bank i
To observe true shocks
To bid for variable-rate auction
Bid amount: A_t^*
Bid rate: $\omega\{(1 + i_t)S_t/f_t\}$
To receive A_t^* from the auction
To make U.S. dollar loans to retailers
To return their proceeds to the central bank
$\omega\{(1 + i_t)S_t/f_t\}$ as promised in the auction

$\Downarrow$

Final Stage
To buy back domestic currencies
$(M_t = S_t(A_t^* - F_t^*))$
at the initial stage's exchange rate
To pay the contractual interest
$z_t(A_t^* - F_t^*)$

a supply level of A_t^*. The random variable ω is independently and identically distributed across time and across local markets with a continuous and once-differentiable cumulative probability distribution function, $\Upsilon(\omega)$, over a non-negative support and also this expected value is equal to one, $E[\omega] = 1$. There are "news shocks" about realized values of these market-specific liquidity shocks before their true realization. The information content of "news shocks" reflects the capability of the foreign central bank to gather information about correct liquidity conditions of local financial markets in its jurisdiction.

The information structure of the model can be described as follows. It is important to determine whether the foreign central bank has some information about the realization of U.S. dollar liquidity shocks when it enters into the contract with the Federal Reserve. The reason for this one is that the contractual interest rates of swap lines become stop-out rates of auctions used to distribute U.S. dollars, while swap agreements between central banks should be determined before auctions. In fact, if the central bank's drawings from its swap lines are not their normal routine works, the central bank should be able to determine in each period whether or not it would use its swap line.

For this reason, it is assumed that there are "news shocks" about realized values of market-specific idiosyncratic liquidity shocks. These "news shocks" have the same cumulative probability distribution as those of idiosyncratic shocks. The foreign central bank knows when it enters in the swap contract with the Federal Reserve and also believes that their knowledge about "news shocks" will hold true. Hence, the foreign central bank believes that it has the full information about the realization of each local market's risk denoted by ω. So as mentioned above, the information content of "news shocks" reflects the capability of the foreign central bank to gather information about correct liquidity conditions of local financial markets in its jurisdiction. However, the Federal Reserve does not observe the realization of "news shocks" but knows their probability distributions denoted by $\Upsilon(\omega)$.

Definition 3.2 (Variable-Rate Auction) The foreign central bank announces a "variable-rate auction" to distribute an amount of U.S. dollars $(1-\Upsilon(\bar{\omega}_t))A_t^*$. Each wholesale bank whose market's liquidity shock is ω bids $\omega\{(1+i_t)S_t/f_t\}$ for A_t^* in the variable-rate auction provided by the foreign central bank. The stop-out rate for the auction is set by satisfying the following condition:

$$\bar{\omega}_t\{(1+i_t)S_t/f_t\}A_t^* = (1+z_t)(A_t^* - F_t^*)$$

where z_t is the interest rate at period t for the foreign exchange swap line with the Federal Reserve. If a wholesale bank (whose shock ω is greater than $\bar{\omega}_t$) has a successful bid in the auction, it repays $\omega\{(1+i_t)S_t/f_t\}A_t^*$ to the foreign central bank.

A wholesale bank whose market's idiosyncratic shock is ω sets its bid rate as $\omega\{(1+i_t)S_t/f_t\}$. The foreign central bank then sets the lowest successful bid rate (denoted by $\bar{\omega}_t$) by satisfying the following equation:

$$\bar{\omega}_t\{(1+i_t)S_t/f_t\}A_t^* = (1+z_t)(A_t^* - F_t^*).$$

Hence, the profit of the central bank that can be obtained from the variable-rate auction is given by

$$(1-\Phi(\bar{\omega}_t))\{(1+i_t)S_t/f_t\}A_t^*; \qquad \Phi(\bar{\omega}) = \int_0^{\bar{\omega}} \omega d\Upsilon(\omega) + \bar{\omega}(1-\Upsilon(\bar{\omega})).$$

Only a fraction of wholesale banks have U.S. dollar loans from the central bank's auctions. Those banks who are not successful or do not participate in the central bank's auctions not need to make any repayments to the central bank. Hence, the total amount of U.S. dollars distributed through this auction is $(1-\Upsilon(\bar{\omega}_t))A_t^*$, while the other fraction $\Upsilon(\bar{\omega}_t)A_t^*$ is sold at retail foreign-exchange markets.

It might be possible to take into account other ways of setting the minimum rate in auctions. For example, although it might be possible to consider a rule for setting the minimum rate that makes a zero cash flow of the central bank, this one may not be useful to make a well-defined contract problem for central bank swap lines that address the issue of moral hazard. Hence, I would choose the rule of the minimum rate discussed above.

I now turn to the condition under which private debt contracts between private agents cannot replicate the foreign exchange swap lines discussed above if private international investors cannot verify realized values of idiosyncratic shocks in the portfolios of private financial institutions. It is necessary to take into account this condition because the debt contract developed in Bernanke, Gertler and Gilchrist (1999) can create exactly the same expected profit of the foreign central bank as that of a private agent who participates in the contract with its net worth F_t^*.

In order to show this result, let us note that the debt contract between the foreign central bank and a private international investor leads to a non-default loan rate z_t and a threshold value of the idiosyncratic shock $\bar{\omega}_t$ such that for values of the idiosyncratic shock greater than or equal to $\bar{\omega}_t$. Under the debt contract, the whole realized values of the idiosyncratic shock is partitioned into default and non-default regions. In addition, the private international investors can borrow in the international financial market at a rate of i_t^*.

If the idiosyncratic shock belongs to the non-default region ($\omega \geq \bar{\omega}_t$), the foreign central bank transfers $(1+z_t)(A_t^* - F_t^*)$ to its counter-party and keeps $\omega\{(1+i_t)S_t/f_t\}A_t^*$ - $(1+z_t)(A_t^* - F_t^*)$. In the default region of $\omega < \bar{\omega}_t$, the foreign central bank cannot pay the contractual return and thus declares default. The private investor pays the auditing cost and gets to keep what it finds. The default cost is proportional to the realized revenue: $\mu\omega\{(1+i_t)S_t/f_t\}A_t^*$ where a positive constant μ

is less than one. In the default region, it receives $(1-\mu)\omega\{(1+i_t)S_t/f_t\}A_t^*$. As a result, if a private international investor borrows in the international financial market to invest in the debt contract with the foreign central bank, its expected cash flow under the debt contract can be written as

$$\Pi(\bar{\omega}_t, A_t^*; F_t^*) = \Psi(\bar{\omega}_t)\{(1+i_t)S_t/f_t\}A_t^* - (1+i_t^*)(A_t^* - F_t^*); \quad \Psi(\bar{\omega}) = \Phi(\bar{\omega}) - \mu\int_0^{\bar{\omega}} \omega d\Upsilon(\omega)$$

where $\Pi(\bar{\omega}_t, A_t^*; F_t^*)$ is the expected cash flow of the private international investor under the debt contract with the foreign central bank. As a result, the optimal debt contract between a private international investor and the foreign central bank can be summarized as follows.

Definition 3.3 (Debt Contract) Let us suppose that private international investors can borrow in the international financial market at a rate of i_t^*. Under a debt contract with a private international investor, the foreign central bank solves the following optimization problem:

$$\max_{\bar{\omega}_t, A_t^*}(1-\Phi(\bar{\omega}_t))\{(1+i_t)S_t/f_t\}A_t^* \quad \text{s.t.} \quad \Pi(\bar{\omega}_t, A_t^*; F_t^*) \geq 0$$

taking as given $\{(1+i_t)S_t/f_t\}$ and F_t^*. The equality of the constraint holds with free entries of private investors for debt contracts.

In particular, the debt contract defined above is the one that would have been available when the Federal Reserve and local wholesale banks could make debt contracts without any intermediation of the foreign central bank. In this case, swap lines could have proceeded without any explicit role of the foreign central bank. It is also possible to have debt contracts between the foreign central bank and local wholesale banks, while the foreign central bank transfer a contractual interest to the Federal Reserve. But any debt contracts defined above cannot bring about the role of variable-rate auctions in central-bank swap lines. Hence, swap contracts discussed in this paper is differentiated from debt contracts even with the fact that the two types of contracts have an identical form of optimization problem.

I now turn to the characterization of central-bank liquidity swap contracts. In particular, it would be natural to require that a formal definition of central-bank currency swap contract should include the following features to reflect central bank liquidity swap lines observed during the Global Financial Crisis of 2008-2009:

1. Repurchase agreement at the same exchange rate

2. Variable-rate auction between the foreign central bank and wholesale banks

3. Payment of contractual interest by the foreign central bank for its borrowing in swap lines

4. Absence of liquidity provider's bearing of credit risk

Given this characterization of central-bank swap lines, a solution to the following optimization problem can be used to satisfy the requirement for the characterization of central-bank swap lines.

Definition 3.4 (Central-Bank Swap Contract) Let us assume that i_t^* represents the Federal Reserve' opportunity cost for its lending to the foreign central bank. Under a central-bank currency swap contract, the foreign central bank solves the following optimization problem:

$$\max_{\bar{\omega}_t, A_t^*}(1-\Phi(\bar{\omega}_t))\{(1+i_t)S_t/f_t\}A_t^* \quad \text{s.t.} \quad \underline{\Pi}_t \leq \Pi(\bar{\omega}_t, A_t^*; F_t^*) \leq 0 \;\; \text{and} \;\; z_t \; > \; i_t^*$$

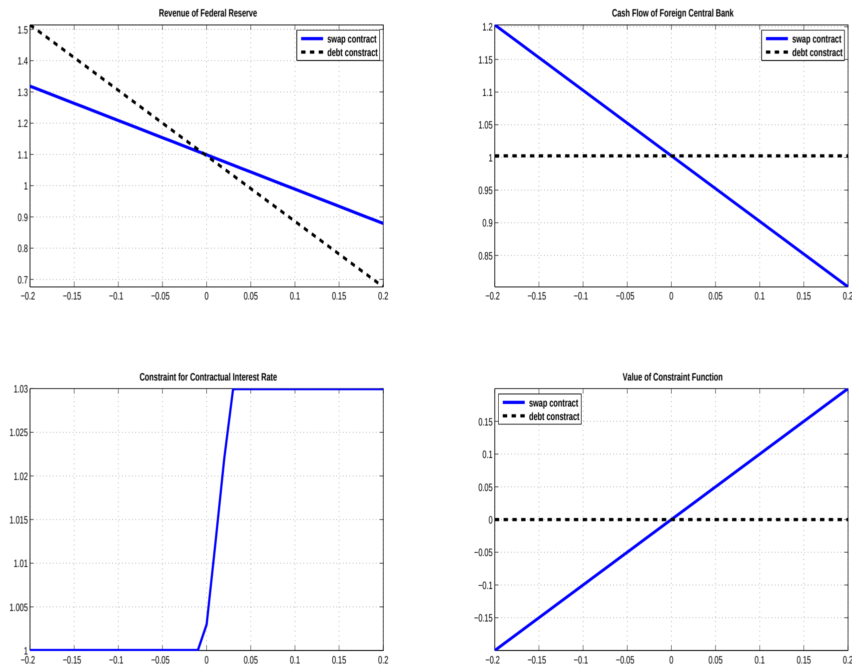
taking as given $\{(1+i_t)S_t/f_t\}$, F_t^*, and a finite negative value $\underline{\Pi}_t$.

I now turn to the discussion of the reasons why such constraints should be included to define a swap problem. The requirement of $z_t > i_t^*$ can be regarded as the Federal Reserves's participation constraint in the central-bank swap contract given a level of the foreign central bank's borrowing, because this condition means that the Federal Reserve's revenue under a debt contract with free entries of private investors for liquidity provision is dominated by that under the swap contract defined above. The condition of $\Pi(\bar{\omega}_t, A_t^*; F_t^*) \leq 0$ (along with $z_t > i_t^*$ in the case of the equality) can be interpreted as the one that eliminates the possibility of the private provision of liquidity under a debt contact with the same conditions that can be attained under a central-bank currency swap contract. It also serves as a necessary condition for the Federal Reserve to choose a swap contract rather than a debt contract.

In other words, the constraint of $\Pi(\bar{\omega}_t, A_t^*; F_t^*) \leq 0$ guarantees that the Federal Reserve does not participate in a debt contract (with free entries of private lenders) with the same form of objective function as that of the swap contract. This consideration is legitimate because of asymmetric information between the Federal Reserve and foreign central banks. In addition, since a lower value of the constraint function leads to a higher value of the objective function but a lower value of the Federal Reserve's revenue, it is not unreasonable to assume that there is a lower bound for the constant function that makes the Federal Reserve participate in the contract, denoted by $\underline{\Pi}_t$ in the definition of a central-bank swap contract. So I have rationalized the reason why it is necessary to include the constraints for the central-bank swap contract problem shown above.

Example 3.1 (A Numerical Example) Having described optimization problems of debt and swap contracts, I provide a numerical example for the model that might help readers to understand important features of the model. The distribution of market-specific liquidity shocks follows a log-normal distribution: $\log\omega \sim \mathcal{N}(-0.5\sigma^2, \sigma^2)$ where I set $\sigma = 0.28$. In addition, the size of auditing costs that would occur with bankruptcies of retail banks is set to be proportional to the realized value of their portfolios whose proportionality constant is set to be $\mu = 0.12$, following

Figure 2: Cash Flows of Central Banks under Swap and Debt Contracts



Note: The upper panels of this figure displays cash flows of the Federal Reserve and foreign central banks under debt and swap contracts as the value of participation constraint varies staring from -0.2 to 0.2. The left lower panel shows that the contractual interest rate under the swap contract is lower than the opportunity cost for the Federal Reserve.

the calibration of Bernanke, Gertler and Gilchrist (1999). The spread of covered interest parity is simply set to be 5 percent in this example. Figure 2 compares cash flows of central banks under swap and debt contracts. The upper panels of this figure displays cash flows of the Federal Reserve and foreign central banks under debt and swap contracts as the value of participation constraint varies staring from -0.2 to 0.2. The left lower panel shows that the contractual interest rate under the swap contract is lower than the opportunity cost for the Federal Reserve. The upper right panel indicates that the cash flow of the foreign central bank is higher under the swap contract than that under the debt contract when $\epsilon < 0$. The lower left panel also shows that the constraint of $z_t > i_t^*$ is binding at around $\epsilon =$ -0.01, while the value of z_t rises when the value of ϵ increases. In particular, $z_t > i_t^*$ at $\epsilon = 0$. As a result, the left lower panel implies that the Federal Reserve will choose the swap contract when $\epsilon = 0$, even when a debt contract is available.

I now turn to a characterization of the optimal swap contract by using its definition spelled out above. It would be crucial to make the following assumption in order to guarantee the existence of solutions to both debt and swap contracts defined above.

Table 4: Expected Payoffs of Central Banks in Debt and Swap Contracts

	Federal Reserve	Foreign Central Banks	Participation Constraints
Central Bank Swap Contract	$(1+z_t)B_t^*$ Bear No Credit Risk	$(1-\Phi(\bar{\omega}_t))(1+i_t)S_t f_t^{-1} A_t^*$	$\underline{\Pi}_t \leq \Pi(\bar{\omega}_t, A_t^*; F_t^*) \leq 0$ $z_t > i_t^*$
Debt Contract	$(1+i_t^*)B_t^*$ Bear Credit Risk	$(1-\Phi(\bar{\omega}_t))(1+i_t)S_t f_t^{-1} A_t^*$	$\Pi(\bar{\omega}_t, A_t^*; F_t^*) = 0$

Note: In the central bank swap contract, the constraint of $\Pi(\bar{\omega}_t, A_t^*; F_t^*) \leq 0$ in this table guarantees that a debt contract (with free entries of private lenders) is not superior to the swap contract for the Federal Reserve.

Assumption 3.1 There is a unique solution to the debt contract problem with free entries of private liquidity providers: $\Pi(\bar{\omega}_t, A_t^*; F_t^*) = 0$. In this case, the contractual interest denoted by z_t is greater than the opportunity cost of liquidity providers (including the Federal Reserve) denoted by i_t^*: $z_t > i_t^*$.

A detailed discussion of a set of sufficient conditions that make this assumption hold true will be provided in the appendix. An important assumption is that the average gross return on the foreign central bank's lending to domestic financial institutions (denoted by $(1+i_t)S_t/f_t$) is greater than the Federal Reserve's gross opportunity cost (denoted by $1+i_t^*$). In other words, the gross return from a one-dollar investment on bonds available in local domestic markets with the shortage of U.S. dollars dominates that of the Federal Reserve's opportunity cost. If the Federal Reserve's opportunity cost of its own lending to foreign central banks is the lowest interest rate available in the world, it might not be unrealistic to assume that the Federal Reserve's opportunity cost should be lower than interest rates of financial markets with liquidity problems. However, this assumption is equivalently converted into a formal assumption that, given a set of values for i_t, i_t^*, and S_t, there is an upper bound on potentially possible values of f_t to make the following strict inequality hold: $f_t < (1+i_t)S_t/(1+i_t^*)$.

Assumption 3.2 The value of the constraint function is denoted by $\epsilon_t = \Pi(\bar{\omega}_t, A_t^*; F_t^*)$. The contractual interest at t denoted by z_t is continuously differentiable with respect to ϵ_t. In this case, $\partial z_t/\partial \epsilon_t < 0$.

If this assumption holds true, the contractual interest under a swap contract decreases with the value of the constraint function $\Pi(\bar{\omega}_t, A_t^*; F_t^*)$. A detailed discussion of a set of sufficient conditions that make **assumption 3.2** hold true will be provided in the appendix. The immediate implication of this assumption is that the highest level of the contractual interest rate can be attained with $\Pi(\bar{\omega}_t, A_t^*; F_t^*) = 0$. In this case, **assumption 3.1** discussed above guarantees that the contractual interest is greater than the opportunity cost of liquidity providers: $z_t > i_t^*$. The discussions made up until now are summarized in the following proposition.

Proposition 3.1 Let us suppose that assumptions 3.1 and 3.2 are satisfied. Then the following statements hold true.

1. Let us suppose that $\Pi(\bar{\omega}_t, A_t^*; F_t^*) = 0$ holds. In this case, the Federal Reserve prefers a swap contract to a debt contract.

2. The maximum value of z_t is attained with $\Pi(\bar{\omega}_t, A_t^*; F_t^*) = 0$ (with $z_t > i_t^*$).

3. In the case of $\Pi(\bar{\omega}_t, A_t^*; F_t^*) = 0$, the total amount of foreign-exchange reserves available at period t under a swap contract can be written as

$$A_t^* = \phi(\frac{(1+i_t)S_t}{(1+i_t^*)f_t})F_t^*$$

where function $\phi(x)$ is an increasing function of x.

A detailed discussion of formal proofs will be provided in the appendix. The most important implication of this proposition is that both debt and swap contracts can be characterized by the same optimization problem, while objective and constraint have different meanings. In particular, one might wonder the reason why the optimal swap contract can guarantee a risk-less contractual interest whereas the corresponding debt contract cannot. The main reason behind this result is that wholesale banks transfer their realized gross returns (above the contractual payment to the Federal Reserve) to the central bank under the swap contract, whereas the debt contract requires them to transfer only contractual gross returns that are below their realized ones. It also should be noted that the absence of the information asymmetry between the foreign central bank and wholesale banks can make variable-rate auctions available, while the information asymmetry between the Federal Reserve and the foreign central bank exists.

It is now worthwhile to discuss the role of the spread of "covered interest parity" in this financial contract, $\{(1+i_t)S_t/(1+i_t^*)f_t\}$. It should be noted that this equilibrium condition is essentially the same as that of Bernanke, Gertler and Gilchrist (1999). In their model with asymmetric information, it is the case that raising funds from lenders is virtually always more expensive than internal finance.

The reason behind this positive external finance premium is that outside lenders should take some costs of evaluating borrowers' prospects and monitoring their actions. But I should emphasize that, in this model, the Federal Reserve as the lender does not bear any credit costs. In contrast, the foreign central bank bears credit risk associated with the lending of U.S. dollars to local financial institutions. However, it still holds true in this model that the external finance premium paid by a borrower should depend inversely on the strength of the borrower's financial position, measured by the size of international reserves in this model, while the spread of "covered interest parity" can be interpreted as the external finance premium for those institutions via the intermediation of their central banks.[5]

The financial capacity of international reserves would emerge in this framework discussed so far. Specifically, international reserves act as collateral for drawings of swap lines. Hence, a country's own international reserves can create a multiplier effect for the amount of U.S. dollars available to maintain the stability of domestic financial markets and the foreign-exchange market. This multiplier effect is thus regarded as the stabilizing power of international reserves that could arise with an efficient use of central bank swap lines. The third bullet point of **Proposition** 3.1 is directly related to the magnitude of the international capacity of international reserves. In sum, given a level of international reserves I_t in period t, this proposition finds that a country's maximum capacity of U.S. dollars denoted by A_t^* can be written as

$$A_t^* = \phi(\frac{(1+i_t)S_t}{(1+i_t^*)f_t})I_t^*.$$

4 Discussion on the Role of Central-Bank Swap Lines

In this section, I discuss how to put the model of central bank swap lines into a small-open DSGE model and also summarize the role of central bank swap lines in the small open economy that is inflicted by adverse balance-of-payments shocks. Above all, the presence of central bank swap lines requires that the government should record changes in the central bank's debt position and revenues from its auction for its drawings of swap lines. The government's flow budget constraint at period t can be written as

$$M_t - M_{t-1} + \frac{B_{H,t}}{1+i_{H,t}} + (M_t^{CB} - M_{t-1}^{CB} + S_t V_t^*) = B_{H,t-1} + S_t(I_t^* - I_{t-1}^*) + P_t(T_t - G_t)$$

where M_t^{CB} represents the nominal amount at period t of the central bank's collateral for its swap lines with the Federal Reserve and V_t^* is the dollar value of central bank's revenues from its auctions.

[5]In models with carry trades, this spread of "covered interest parity" can be interpreted as the carry trade margin earned by a currency trader who borrows U.S. dollars in the international financial market and covert them into a local currency to invest in the certificates of deposit in that country. When the spread of "covered interest parity" is high, the return to the carry trade (long position in the local currency and short in dollars) yields a high return.

The evolution equation of international reserves is also given by

$$I_t^* = I_{t-1}^* - D_{F,t}/S_t + (M_t^{CB} - M_{t-1}^{CB})/S_t + V_t^*$$

It should be noted that there is limitation in the role of central bank swap lines to defend the economy from adverse balance-of-payments shocks. However, when the shock is not large enough, the use of central-bank swap lines helps eliminate the case in which households and firms begin to believe that a future change in the asset market structure will take place. In order to show this result, let us assume that $\bar{D}_{F,t}$ represents the maximum of the excess demand of U.S. dollars at period t that can be accommodated with the government's holding of international reserves: $\bar{D}_{F,t} = S_t I_{t-1}^*$. If the excess demand for U.S. dollars reaches this level, the balance-of-payments equation becomes an effective equilibrium condition when households and firms begin to believe that the economy will run into financial autarky. But if $\bar{D}_{F,t} < S_t A_t^*$ where $A_t^* = \phi(\frac{(1+i_t)S_t}{(1+i_t^*)f_t})I_t^*$, the evolution equation of international reserves in the presence of central-bank swap lines can be written as

$$I_t^* = (1 - \phi(\frac{(1+i_t)S_t}{(1+i_t^*)f_t})^{-1})I_{t-1}^* + V_t^*$$

where V_t^* is the central bank's revenue from its auction with local financial institutions. To the extent which $\bar{D}_{F,t} < S_t A_t^*$ holds, there is a positive amount of international reserves available in the next period. The use of central-bank swap lines therefore helps eliminate the case in which households and firms begin to believe that a future change in the asset market structure will take place, so long as $\bar{D}_{F,t} < S_t A_t^*$ holds. An implication of this result is that the availability of central-bank swap lines can play a role as a complement of international reserves but not a substitute for international reserves. The reason for this one is that the borrowing of U.S. dollars from the Federal Reserve is not available unless the home government holds its own international reserves in the model of this paper.

5 Conclusion

In this paper, I have investigated business-cycles implications of a small-open macroeconomic DSGE model with central-bank swap lines for the financial crisis and the relation between international reserves and central-bank swap lines. The model of central-bank swap-lines analyzed in this paper has been focused on the issue of moral hazard that can arise with the liquidity provision of the Federal Reserve through its swap lines with other central banks. In particular, there are short-run and long-run channels through which the behavior of international reserves and also the credible long-term stance of the monetary policy can affect the expectation formation of agents. It has been shown that these channels are important factors to mitigate the potential adverse effects of the balance-of-payments shocks signaling future changes in the asset market structure.

I would mention a couple of issues that might be worthy of future research. First, while the model's main concern is bilateral contracts, it would be interesting to extend the current model to the analysis of multi-lateral contracts. Second, the absence of information asymmetry between the foreign central bank and wholesale banks in its jurisdiction makes it possible to use "variable-rate auctions" in which different loan rates are submitted by different wholesale banks. However, once the information problem comes in the picture, it would be more desirable to make debt contracts between the foreign central bank and wholesale banks. Even with debt contracts between the foreign central bank and wholesale banks, it is possible to deliver a risk-less contractual interest payment to the Federal Reserve. Hence, it would be interesting to analyze the optimal mechanism design for central bank swap lines as a future research.

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